VECTOR PRODUCT OF VECTORS

OBJECTIVES

1. **If** $\bar{a}=2\bar{i}+\bar{j}-\bar{k}$ **, and** $\bar{b}=3\bar{i}-\bar{j}+\bar{k}$ **then** $|\bar{a}\times\bar{b}|=$

1) 5   2) $5\sqrt{2}$ 3) 6   4) $6\sqrt{2}$

2. **If** $|\bar{a}|=2, |\bar{b}|=3$ **and** $(\bar{a},\bar{b})=\frac{\pi}{6}$**, then** $|\bar{a}\times\bar{b}|^2$ **is equal to**

1) 6   2) 8   3) 9   4) 7

3. **The value of** $\bar{a}\times(\bar{b}+\bar{c})+\bar{b}\times(\bar{c}+\bar{a})+\bar{c}\times(\bar{a}+\bar{b})$ **is**

1) $2\bar{a}$ 2) $\bar{0}$ 3) $\bar{b}$ 4) $\bar{c}$

4. **Sine of the angle between** $(\bar{i}+\bar{j})$ **and** $(\bar{j}+\bar{k})$ **is**

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\frac{1}{\sqrt{2}}$

5. **If** $|\bar{a}|$ **= 2** $|\bar{b}|$ **= 7 and** $(\bar{a},\bar{b})$ **=**

1) 30°   2) 60°   3) 45°   4) 75°

6. **If** $(2\bar{i}+4\bar{j}+2\bar{k})\times(2\bar{i}+x\bar{j}+5\bar{k})=16\bar{i}+6\bar{j}-2x\bar{k}$

1) 2   2) -2   3) 0   4) 1

7. **If A = (t,1,2t), B=(3,1,2) and** $\bar{p}=4\bar{i}-\bar{j}+3\bar{k}$ **such that** $\overline{AB}\times\bar{p}=6\bar{j}+9\bar{j}-5\bar{k}$**, then the value of t is**

1) -2   2) 2   3) -3   4) 3

8. **The area of the parallelogram whose adjacent sides are** $3\bar{i}+2\bar{j}+\bar{k}$ **and** $3\bar{i}+\bar{k}$ **is. [‘90]**

1) $3\sqrt{10}$ sq. Units   2) $2\sqrt{10}$ sq. Units

3) $4\sqrt{10}$ sq. Units   4) $5\sqrt{10}$ sq. Unit

9. The area of the parallelogram having diagonals $\bar{a} = 3\bar{i} + \bar{j} - 2\bar{k}$ **and** $\bar{b} = \bar{i} - 3\bar{j} + 4\bar{k}$ **is**

1) $5\sqrt{3}$ sq. Units 2) $4\sqrt{3}$ sq. Units 3) $6\sqrt{3}$ sq. Units 4) $3\sqrt{3}$ sq. Units

10. The vector area of the triangle whose adjacent sides are $2\bar{i} + 3\bar{j}$ **and** $-2\bar{i} + 4\bar{j}$ **is**

1) $7\bar{i}$ 2) $7\bar{j}$

3) $7\bar{k}$ 4) $7\bar{i} + 7\bar{j} + 7\bar{k}$

11. The area of the triangle whose two sides are given by $2\bar{i} - 7\bar{j} + \bar{k}$ **and** $4\bar{j} - 3\bar{k}$ **is**

1) 17 2) $\frac{17}{2}$

3) $\frac{17}{4}$ 4) $\frac{1}{2}\sqrt{389}$

12. The area of the triangle whose vertices are (1,0,0), (0,1,0) and (0,0,1) is

1) $\sqrt{3}$ sq. Units 2) $\frac{1}{2}\sqrt{3}$ sq.units

3) 3 sq. Units 4) $\frac{3}{2}$ sq. Units

13. The area of the triangle formed by the points whose position vectors are $3\bar{i} + \bar{j}, 5\bar{i} + 2\bar{j} + \bar{k}$ **and** $\bar{i} - 2\bar{j} + 3\bar{k}$ **is.**

1) $\sqrt{23}$ sq. Units 2) $\sqrt{21}$ sq. Units

3) $\sqrt{29}$ sq. units 4) $\sqrt{33}$ sq. Units

14. If $\bar{a}, \bar{b}, \bar{c}$ **are the position vectors of A,B,C of** Δ**ABC, then** $(\bar{a} \times \bar{b}) + (\bar{b} \times \bar{c}) + (\bar{c} \times \bar{a})$ **is equal to**

1) $\frac{1}{2}(\Delta ABC)$ 2) $2(\Delta ABC)$

3) $3(\Delta ABC)$ 4) $\frac{1}{3}(\Delta ABC)$

15. **A Vector which is normal to both the vectors $\bar{i}+2\bar{j}+3\bar{k}$ and $-\bar{i}+2\bar{j}+\bar{k}$ is**

1) $\bar{i}+\bar{j}-\bar{k}$ 2) $-4(\bar{i}+\bar{j}+\bar{k})$ 3) $-4(\bar{i}+\bar{j}-\bar{k})$ 4) $4(\bar{i}+\bar{j}+\bar{k})$

16. **A unit vector perpendicular to each of the vectors $\bar{i}+2\bar{j}-\bar{k}$ and $2\bar{i}+3\bar{j}+\bar{k}$ is**

1) $\frac{1}{35}(5\bar{i}+3\bar{j}+\bar{k})$ 2) $\frac{1}{\sqrt{35}}(5\bar{i}-3\bar{j}-\bar{k})$

3) $\frac{1}{\sqrt{35}}(5\bar{i}+3\bar{j}+\bar{k})$ 4) $\frac{1}{35}(5\bar{i}-3\bar{j}-\bar{k})$

17. **The unit vector perpendicular to each of the vectors $2\bar{i}-\bar{j}-\bar{k}$ and $3\bar{i}+4\bar{j}-\bar{k}$ is ['91]**

1) $-3\bar{i}+5\bar{j}+11\bar{k}$ 2) $\frac{1}{155}(-3\bar{i}+5\bar{j}+11\bar{k})$

3) $\frac{1}{155}(-3\bar{i}+5\bar{j}+11\bar{k})$ 4) $\frac{1}{155}(3\bar{i}+5\bar{j}+11\bar{k})$

18. **If $\bar{a}=-\bar{i}+\bar{j}+\bar{k}$ and $\bar{b}=\bar{i}-\bar{j}+\bar{k}$, then a unit vector perpendicular to $\bar{a}$ and $\bar{b}$ is**

1) $\frac{1}{\sqrt{2}}(\bar{i}-\bar{j})$ 2) $\bar{k}$

3) $\frac{1}{\sqrt{2}}(\bar{i}+\bar{j})$ 4) $\frac{1}{\sqrt{2}}(\bar{j}+\bar{j})$

19. **A vector of magnitude $\sqrt{6}$ which is perpendicular to both the vectors $2\bar{i}+\bar{j}+\bar{k}$ is**

1) $\pm(2\bar{i}+\bar{j}+\bar{k})$ 2) $\pm(2\bar{i}-\bar{j}+\bar{k})$

3) $\pm(2\bar{i}-\bar{j}-\bar{k})$ 4) $\pm(3\bar{i}-\bar{j}-2\bar{k})$

20. **If $\bar{c}$ is a unit vector perpendicular to the two vectors $\bar{a}$ and $\bar{b}$, then the second unit vector perpendicular to $\bar{a}$ and $\bar{b}$ is**

1) $\bar{c}\times\bar{a}$ 2) $\bar{c}\times\bar{b}$ 3) $-\bar{c}$ 4) $-2\bar{c}$

21. The value of $\left(\bar{a}\times\bar{b}\right)^2+\left(\bar{a}.\bar{b}\right)^2$ =

1) ab 2) a^2b^2 3) a+b 4) a-b

22. If $\bar{a}=2\bar{i}+2\bar{j}+\bar{k}$**,** $\bar{a}.\bar{b}$ **= 14 and** $\bar{a}\times\bar{b}=3\bar{i}+\bar{j}-8\bar{k}$**, then** $\bar{b}$ =

1) $5\bar{i}-\bar{j}-2\bar{k}$ **2)** $5\bar{i}+\bar{j}+2\bar{k}$

3) $5\bar{i}+\bar{j}+2\bar{k}$ 4) $5\bar{i}-\bar{j}+\bar{k}$

23. If $\left(\bar{a}\times\bar{b}\right)^2+\left(\bar{a}.\bar{b}\right)^2$ **= 144 and** $\left|\bar{a}\right|$ **= 4, then** $\left|\bar{b}\right|$ =

1) 16 2) 8 3) 3 4) 12

24. If $\left(\bar{a}\times\bar{b}\right)^2=\lambda-\left(\bar{a}.\bar{b}\right)^2$ **where** $\left|\bar{a}\right|$ **= a and** $\left|\bar{b}\right|$**=b, then the value of** λ **is**

1) ab 2) a^2b 3) ab^2 4)a^2b^2

25. If $\left|\bar{a}\right|=2\left|\bar{b}\right|=5$ **and** $\left|\bar{a}\times\bar{b}\right|=8$**, then** $\bar{a}.\bar{b}$ =

1) 4 2) 5 3) 6 4) 7

26. If $\bar{a}=2\bar{i}+\bar{j}-\bar{k},\ \bar{b}=-\bar{i}+2\bar{j}-4\bar{k}$ **and** $\bar{c}=\bar{i}+\bar{j}+\bar{k}$**, then** $\left|\left(\bar{a}\times\bar{b}\right).\left(\bar{a}\times\bar{b}\right)\right|$ =

1) 26 2) 13 3) 39 4) 15

27. If $\bar{i},\bar{j},\bar{k}$ **are unit orthonormal vectors and** $\bar{a}$ **is unit vector such that** $\bar{a}\times\bar{i}=\bar{j}$**, then** $\bar{a}.\bar{i}$ is

1) 0 2) 1

3) -1 4) Arbitrary Scalar

28. If $\bar{a}+2\bar{b}+4\bar{c}=\bar{o}$**, then** $\left(\bar{a}\times\bar{b}\right)+\left(\bar{b}\times\bar{c}\right)+\left(\bar{c}\times\bar{a}\right)=$

1) $8\left(\bar{b}\times\bar{c}\right)$ 2) $7\left(\bar{b}\times\bar{c}\right)$

3) $4\left(\bar{b}\times\bar{c}\right)$ 4) $5\left(\bar{b}\times\bar{c}\right)$

29. If $\bar{a}\times\bar{b}=\bar{c}$ **and** $\bar{b}\times\bar{c}=\bar{a}$**, then**

1) a = 1, b = 1 2) c = 1, a =1 3) b = 2, c = 2a 4) b = 1, $\bar{c}$ = $\bar{a}$

30. **If $\bar{a}.\bar{b}$ = $\bar{a}.\bar{c}, \bar{a}\times\bar{b}=\bar{a}\times\bar{c}$ and $\bar{a}\neq\bar{o}$, then**

1) $\bar{a}=\bar{b}$ 2) $\bar{a}=\bar{c}$

3) $\bar{b}=\bar{c}$ 4) $\bar{a}=2\bar{b}$

31. **A = $\bar{a}$, B = $\bar{b}$ and C = $\bar{c}$ are the vertices of Δ ABC, then the perpendicular distance from A on BC is**

1) $\left|(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})+(\bar{a}\times\bar{b})\right|\div\left|\bar{b}-\bar{a}\right|$

2) $\left|(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})+(\bar{a}\times\bar{b})\right|\div\left|\bar{c}-\bar{a}\right|$

3) $\left|(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})+(\bar{a}\times\bar{b})\right|\div\left|\bar{c}-\bar{b}\right|$

4) $\left|(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})+(\bar{a}\times\bar{b})\right|\div\left|\bar{a}+\bar{b}\right|$

32. **Given $\bar{a}=\bar{i}+\bar{j}+\bar{k}, \bar{b}=-\bar{i}+2\bar{j}+\bar{k}$ and $\bar{c}=-\bar{i}+2\bar{j}-\bar{k}$. A unit vector perpendicular to both $\bar{a}+\bar{b}$ and $\bar{b}+\bar{c}$ is**

1) $\bar{i}$ 2) $\bar{j}$

3) $\bar{k}$ 4) $\frac{1}{\sqrt{3}}(\bar{i}+\bar{j}+\bar{k})$

33. **If $\bar{a}\times\bar{b}=\bar{c}\times\bar{d}$ and $\bar{a}\times\bar{c}=\bar{b}\times\bar{d}$ where $\bar{a}\neq\bar{d}, \bar{b}\neq\bar{d}$, then $\bar{a}-\bar{d}$ is**

1) Parallel to $\bar{b}-\bar{c}$

2) Perpendicular to $\bar{b}-\bar{c}$

3) Inclined at an angle other then $\frac{\pi}{2}$

34 **ABCD is a quadrilateral with $\overline{AB}=\bar{a}, \overline{AD}=\bar{b}, \overline{AC}=2\bar{a}+3\bar{b}$. If the area is p times the area of the parallelogram with AB , Ad as adjacent sides, then p is equal to**

1) 5 2) $\frac{5}{2}$ 3) 1 4) $\frac{1}{2}$

35. **Three vectors** $\bar{a},\bar{b},\bar{c}$ **are such that** $\bar{a}\times\bar{b}=2\bar{a}\times\bar{c}$, $|\bar{a}|=|\bar{c}|=1$ **and** $|\bar{b}|=4$**. If the angle between** $\bar{b}$ **and** $\bar{c}$ **is** $\cos^{-1}\left(\frac{1}{4}\right)$**, then** $\bar{b}-2\bar{c}$ **is equal to**

1) $\pm4\bar{a}$ 2) $\pm3\bar{a}$ 3) $\pm5\bar{a}$ 4) $\pm2\bar{a}$

36. $\bar{r}\times\bar{a}=\bar{b}\times\bar{a},\bar{r}\times\bar{b}=\bar{a}\times\bar{b}$**,** $\bar{a}\neq\bar{o},\bar{b}\neq\bar{o},\bar{b}\neq\lambda\bar{b},\bar{a}$ **is not perpendicular to** $\bar{b}\Rightarrow\bar{r}=$

1) $\bar{a}-\bar{b}$ 2) $\bar{a}+\bar{b}$

3) $\bar{a}\times\bar{b}+\bar{b}$ 4) $\bar{a}\times\bar{b}-\bar{b}$

37. **Let** $\bar{a},\bar{b},\bar{c}$ **be unit vectors. Suppose** $\bar{a}.\bar{b}=\bar{a}.\bar{c}=0$ **and the angle between** $\bar{b}$ **and** $\bar{c}$ **is** $\frac{\pi}{6}$**. Then** $\bar{a}$ **=**

1) $\pm\left(\bar{b}\times\bar{c}\right)$ 2) $\pm2\left(\bar{b}\times\bar{c}\right)$ 3) $\pm3\left(\bar{b}\times\bar{c}\right)$ 4) $\pm4\left(\bar{b}\times\bar{c}\right)$

38. **If** $\bar{a}$ **and** $\bar{b}$ **are not perpendicular to each other and** $\bar{r}\times\bar{b}=\bar{c}\times\bar{b}$ $\bar{r}.\bar{a}=0$**, then** $\bar{r}$ **=**

1) $\bar{c}+\frac{\bar{c}.\bar{a}}{\bar{b}.\bar{a}}\bar{b}$ 2) $\bar{b}-\frac{\bar{c}.\bar{a}}{\bar{b}.\bar{a}}\bar{c}$ 3) $\bar{c}-\frac{\bar{c}.\bar{a}}{\bar{b}.\bar{a}}\bar{b}$ 4) $\bar{b}+\frac{\bar{c}.\bar{a}}{\bar{b}.\bar{a}}\bar{c}$

39. **If** $\bar{u}=\bar{a}-\bar{b},\bar{\upsilon}=\bar{a}+\bar{b}$ **and** $|\bar{a}|=|\bar{b}|=2$**, then** $|\bar{u}+\bar{\upsilon}|=$

1) $\sqrt{4-\left(\bar{a}.\bar{b}\right)^2}$ 2) $2\sqrt{16-\left(\bar{a}.\bar{b}\right)^2}$ 3) $2\sqrt{4-\left(\bar{a}.\bar{b}\right)^2}$ 4) $\sqrt{16-\left(\bar{a}.\bar{b}\right)^2}$

40. **If** $\bar{a}$ **= (1,1,1),** $\bar{c}$ **= (0,1,-1) are given vectors, then a vector** $\bar{b}$ **satisfying the equations** $\bar{a}\times\bar{b}=\bar{c}$ **and** $\bar{a}.\bar{b}=3$ **is**

1) $\frac{1}{3}\left(5\bar{i}+2\bar{j}-2\bar{k}\right)$ 2) $\frac{1}{3}\left(5\bar{i}-2\bar{j}+2\bar{k}\right)$ 3) $\frac{1}{3}\left(5\bar{i}+2\bar{j}+2\bar{k}\right)$ 4) $\frac{1}{3}\left(-5\bar{i}-2\bar{j}+2\bar{k}\right)$

41. **Let** $\bar{a},\bar{b},\bar{c}$ **be unit vectors such that** $\bar{a},\bar{b}=0=\bar{a}.\bar{c}$**. If the angle between** $\bar{b}$ **and** $\bar{c}$ **is** $\frac{\pi}{6}$**, then** $\bar{a}$ **=**

1) $\pm2\left(\bar{b}\times\bar{c}\right)$ 2) $2\left(\bar{b}\times\bar{c}\right)$ 3) $\pm\frac{1}{2}\left(\bar{b}\times\bar{c}\right)$ 4) $-\frac{1}{2}\left(\bar{b}\times\bar{c}\right)$

42. **If $\bar{a}.\bar{b} = \bar{a}.\bar{c}$ and $\bar{a} \times \bar{b} = \bar{a} \times \bar{c}$, then**

1) $\bar{a}$ is parallel to $\bar{b} - \bar{c}$

2) $\bar{a}$ is perpendicular to $\bar{b} - \bar{c}$

3) Either $\bar{a}$ $\bar{a} = \bar{o}$ or $\bar{b} = \bar{c}$

4) $\bar{b} \neq \bar{c}$

43. **A non-zero vector $\bar{a}$ is parallel to the line of intersection of the plane determined by the vectors $\bar{i}, \bar{i} + \bar{j}$ and the plane determined by the vectors $\bar{i} - \bar{j}, \bar{j} + \bar{k}$. The angle between $\bar{a}$ and the vector $\bar{i} - 2\bar{j} + 2\bar{k}$ is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

44. **ABCDEF is a regular hexagon. If $\overline{AB} = \bar{a}$ and $\overline{BC} = \bar{b}$, then $\overline{AE} \times \overline{AC}$ =**

1) $2\bar{b} \times \bar{a}$ 2) $3(\bar{b} \times \bar{a})$ 3) $3\bar{a} \times \bar{b}$ 4) $3(\bar{a} \times \bar{b})$

45. **Let $\bar{a} = \bar{i} + \bar{j}$ and $\bar{b} = 2\bar{i} - \bar{k}$. Then the point of intersection of the lines $\bar{r} \times \bar{a} = \bar{b} \times \bar{a}$ and $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$ is**

1) (1,-1,-1) 2) (-1,1,1) 3) (3,1,-1) 4) (3,-1,1)

46. **If $\bar{a} \times \bar{b} = \bar{c}$ and $\bar{b} \times \bar{c} = \bar{a}$, then**

1) $|\bar{b}| = 1, |\bar{c}| = |\bar{a}|$ (2) $|\bar{c}| = 1,$ 2) $|\bar{c}| = 1, |\bar{a}| = 1$ 3) $|\bar{b}| = 2, |\bar{b}| = 2|\bar{a}|$ 4) $|\bar{a}| = 1, |\bar{b}| = |\bar{c}|$

47. **If a vector $\bar{r}$ satisfies the equations $\bar{r}.(\bar{i} + 2\bar{j} - 4\bar{k}) = 0$ and $\bar{r} \times (\bar{i} + 2\bar{j} + \bar{k}) = \bar{i} - \bar{k}$, then $|\bar{r}|$ is given by**

1) $\sqrt{13}$ 2) $\sqrt{19}$ 3) $\sqrt{15}$ 4) $\sqrt{17}$

VECTOR PRODUCT OF VECTORS

HINTS AND SOLUTIONS

1. (2)

$\bar{a}\times\bar{b} = -5\bar{j} - 5\bar{k}$

$\Rightarrow |\bar{a}\times\bar{b}| = \sqrt{25+25} = 5\sqrt{2}$

2. (3)

$|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2 \cdot |\bar{b}|^2 \cdot \sin^2(\bar{a},\bar{b})$

$= 4(9)\sin^2 30° = 36(1/4) = 9.$

3. (2)

G.E.= $(\bar{a}\times\bar{b}) + (\bar{a}\times\bar{c}) + (\bar{b}\times\bar{c}) + (\bar{b}\times\bar{a}) + (\bar{c}\times\bar{a}) + (\bar{c}\times\bar{b}) = 0$

$= (\bar{c}\times\bar{b}) - (\bar{c}\times\bar{a}) + (\bar{b}\times\bar{c}) - (\bar{a}\times\bar{b}) + (\bar{c}\times\bar{a}) - (\bar{b}\times\bar{c}) = \bar{O}$

4. (3)

$\sin(\bar{i}+\bar{j}, \bar{j}+\bar{k}) = \dfrac{|(\bar{i}+\bar{j})\times(\bar{j}+\bar{k})}{|\bar{i}+\bar{j}|\cdot|\bar{j}+\bar{k}|}$

$= \dfrac{|\bar{i}-\bar{j}+\bar{k}|}{\sqrt{1+1}\sqrt{1+1}} = \dfrac{\sqrt{3}}{2}.$

5. (1)

Given $|\bar{a}| = 2, |\bar{b}| = 7$, and $\bar{a}\times\bar{b} = 3\bar{i} + 2\bar{j} + 6\bar{k}$

$\sin(\bar{a},\bar{b}) = \dfrac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|}$

$= \dfrac{\sqrt{9+4+36}}{2\times 7} = \dfrac{7}{14} = \dfrac{1}{2}$

$\Rightarrow (\bar{a},\bar{b}) = 30°.$

6. (2)

Given $\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 4 & 2 \\ 2 & -x & 5 \end{vmatrix} = 16\bar{i} - 6\bar{j} + 2xk$

$\Rightarrow (20+2x)\bar{i} - \bar{j}(10-4) + \bar{k}(-2x-8)$

$= 16\bar{i} - 16\bar{j} + 2xk$

$\Rightarrow 20 + 2x = 16 \Rightarrow x = -2$

7. (1)

$AB = (3-t)\bar{i} + 0 + (2-2t)\bar{k},$

$x = 4\bar{i} - \bar{j} + 3\bar{k}$

$\Rightarrow AB \times x = 6\bar{i} + 9\bar{j} - 5\bar{k}$

$\Rightarrow (2-2t)\bar{i} + (3t-9)\bar{j} + (t-3)\bar{k}$

$= 6\bar{i} + 9\bar{j} - 5\bar{k}$

$\Rightarrow 2 - 2t = 6 \Rightarrow 2t = -4 \Rightarrow t = -2.$

8. (2)

Area of the parallelogram

$= |(3\bar{i} + 2\bar{j} + \bar{k}) \times (3\bar{i} + \bar{k})| = \left| \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 2 & 1 \\ 3 & 0 & 1 \end{vmatrix} \right|$

$= |2\bar{i} - 6\bar{k}| = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ sq.units.

9. (1)

Area of the parallelogram =

$\frac{1}{2}|\bar{a} \times \bar{b}| = \frac{1}{2}|-2\bar{i} - 14\bar{j} - 10\bar{k}|$

$= |-\bar{i} - 7\bar{j} - 5\bar{k}|$

$= \sqrt{1+49+25} = \sqrt{75} = 5\sqrt{3}$ sq.units.

10. (3)

Vector area of the given triangle

$$= (1/2)(2\bar{i} + 3\bar{j}) \times (-2\bar{i} + 4\bar{j})$$

$$= (1/2)(14\bar{k}) = 7\bar{k}$$

11. (4)

Area = $\frac{1}{2}|(2\bar{i} - 7\bar{j} + \bar{k}) \times (4\bar{j} - 3\bar{k})|$

$$= \frac{1}{2}|17\bar{i} + 6\bar{j} + 8\bar{k})$$

$$= \frac{1}{2}\sqrt{289+36+64} = \frac{1}{2}\sqrt{389}.$$

12. (2)

Let A = (1,0,0), B = (0,1,0) and C = (0,0,1).

Now AB = (–1, 1, 0) and AC = (–1, 0, 1)

Area of ΔABC = (1/2)|AB × AC|

$$= \left(\frac{1}{2}\right)|\bar{i} + \bar{j} + \bar{k}| = \left(\frac{1}{2}\right)\sqrt{1+1+1}$$

$$= \left(\frac{1}{2}\right)\sqrt{3}\text{ sq.units}$$

13. (3)

Let A, B, C be the given points.

$\overline{BA} = -2\bar{i} - \bar{j} - \bar{k}$, then $\overline{BC} = -4\bar{i} - 4\bar{j} + 2\bar{k}$

$$\overline{BA} \times \overline{BC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2 & -1 & -1 \\ -4 & -4 & 2 \end{vmatrix} = -6\bar{i} + 8\bar{j} - 4\bar{k}$$

Area of ΔABC =

$$\frac{1}{2}|\overline{BA}\times\overline{BC}|=|-3\bar{i}+4\bar{j}-2\bar{k}|$$
$$=\sqrt{9+16+4}=\sqrt{29}\text{ sq.units.}$$

14. (2)

Given $A=\bar{a}, B=\bar{b}, C=\bar{c}$.

Now AB = $\bar{b}-\bar{a}$, $\overline{AC}=\bar{c}-\bar{a}$

$$AB\times AC=(\bar{b}-\bar{a})\times(\bar{c}-\bar{a})$$
$$=\bar{b}\times\bar{c}-\bar{b}\times\bar{a}-\bar{a}\times\bar{c}+\bar{a}\times\bar{a}$$
$$=(\bar{b}\times\bar{c})+(\bar{a}\times\bar{b})+(\bar{c}\times\bar{a})$$

But $(1/2)[(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})]$ represents the vector area of ΔABC.

Hence $(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})=2(\Delta ABC)$

15. (3)

Vector $\perp$ to the given vectors

$$=(\bar{i}+2\bar{j}+3\bar{k})\times(-\bar{i}+2\bar{j}+\bar{k})$$
$$=-4(\bar{i}+\bar{j}-\bar{k})$$

16. (2)

Required unit vector

$$=\frac{(\bar{i}+2\bar{j}-\bar{k})\times(2\bar{i}+3\bar{j}+\bar{k})}{|(\bar{i}+2\bar{j}-\bar{k})\times(2\bar{i}+3\bar{j}+\bar{k})|}$$
$$=\frac{(5\bar{i}-3\bar{j}-\bar{k})}{\sqrt{25+9+1}}=\frac{(5\bar{i}-3\bar{j}-\bar{k})}{\sqrt{35}}$$

17. (3)

Vector perpendicular to each of the vectors $2\bar{i}-\bar{j}+\bar{k}$ and $3\bar{i}+4\bar{j}-\bar{k}$ is

$$(2\bar{i}-\bar{j}+\bar{k})\times(3\bar{i}+4\bar{j}-\bar{k})=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & -1 & 1\\ 3 & 4 & -1\end{vmatrix}$$

$$=3\bar{i}+5\bar{j}+11\bar{k}$$

Unit vector ⊥ to each of the vectors =

$$\frac{-3i+5j+11k}{\sqrt{(-3)^2+(5)^2+(11)^2}}$$

$$=\frac{1}{\sqrt{155}}(-3\bar{i}+5\bar{j}+11\bar{k})$$

18. (3)

$$\bar{a}\times\bar{b}=2\bar{i}\times2\bar{j}$$

Then the unit vector ⊥ to $\bar{a}$ and $\bar{b}$ is $\frac{2\bar{i}+2\bar{j}}{\sqrt{4+4}}=\frac{\bar{i}+\bar{j}}{\sqrt{2}}$.

19. (3)

$$(\bar{i}+\bar{j}+\bar{k})\times(2\bar{i}+\bar{j}+3\bar{k})=2\bar{i}-\bar{j}-\bar{k}.$$

Its unit vector = $\pm\frac{(2\bar{i}-\bar{j}-\bar{k})}{\sqrt{4+1+1}}$

∴ Vector ⊥ to the given vectors and of magnitude $\sqrt{6}$

$$=\frac{\pm\sqrt{6}(2\bar{i}-\bar{j}-\bar{k})}{\sqrt{6}}=\pm(2\bar{i}-\bar{j}-\bar{k}).$$

20. (3)

The unit vector $\perp$ to $\bar{a}$ and $\bar{b}$

$$= \pm\frac{(\bar{a}\times\bar{b})}{|\bar{a}\times\bar{b}|} = \pm c \text{ (given).}$$

If $\bar{c}$ is one unit vector, the other unit vector is $-\bar{c}$.

21. (2)

G.E.= $|a|^2|b|^2 \sin^2\theta + |a|^2|b|^2 \cos^2\theta$

$= |a|^2|b|^2(\cos^2\theta + \sin^2\theta)$

$= |a|^2|b|^2 = a^2b^2$

22. (3)

Let $\bar{b} = x\bar{i} + y\bar{j} + z\bar{k}$

Given $\bar{a}\cdot\bar{b} = 14 \Rightarrow 2x + 2y + z = 14 \ldots(1)$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 2 & 1 \\ x & y & z \end{vmatrix} = 3i + j - 8k \text{ (given)}$$

$\Rightarrow (2z-y)\bar{i} + (x-2z)\bar{j} + (2y-2x)\bar{k}$

$= 3\bar{i} + \bar{j} - 8\bar{k}$

$\Rightarrow 2z - y = 3, x - 2z = 1, 2y - 2x = -8$

Solving (1) and (2) : x = 5, y = 1, z = 2.

$\therefore \bar{b} = 5\bar{i} + \bar{j} + 2\bar{k}$

23. (3)

We have $(\bar{a}\times\bar{b})^2 + (\bar{a}\cdot\bar{b})^2 = \bar{a}^2\bar{b}^2$

(Lagrange's indentify)

$\Rightarrow 144 = 16\bar{b}^2 \; (\because |\bar{a}| = 4)$

$\Rightarrow \bar{b}^2 = 9 \Rightarrow |\bar{b}| = 3$

24. (4)

$(\bar{a}\times\bar{b})^2 + (\bar{a}\cdot\bar{b})^2 = \lambda$

$\Rightarrow (ab\sin\theta\cdot\bar{n})^2 + (ab\cos\theta)^2 = \lambda$

$\Rightarrow a^2b^2\sin^2\theta + a^2b^2\cos^2\theta = \lambda$

$(\because \hat{n}^2 = \hat{n}\cdot\hat{n} = 1) \Rightarrow \lambda = a^2b^2$

25. (3)

Given $|\bar{a}\times\bar{b}| = 8, |\bar{a}| = 2, +|\bar{b}| = 5$

$(\bar{a}\times\bar{b})^2 + (\bar{a}\cdot\bar{b}) = \bar{a}^2\bar{b}^2$

$\therefore |\bar{a}\times\bar{b}|^2 + (\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2$

$\Rightarrow 64 + (\bar{a}\cdot\bar{b})^2 = 4(25)$

$\Rightarrow (\bar{a}\cdot\bar{b})^2 = 36$

$\therefore \bar{a}\cdot\bar{b} = 6$

26. (1)

$\bar{a}\times\bar{b} = (2\bar{i} + \bar{j} - \bar{k})\times(-\bar{i} + 2\bar{j} - 4\bar{k})$

$= 2\bar{i} + 9\bar{j} + 5\bar{k}$ and $\bar{a}\times\bar{c} = 2\bar{i} - 3\bar{j} + \bar{k}$

Now $|(\bar{a}\times\bar{b})\cdot(\bar{a}\times\bar{c})| = |-26| = 26.$

27. (4)

Given $\bar{a}\times\bar{i} = \bar{j} \Rightarrow \bar{a} = x\bar{i} + \bar{k}$

$\therefore \bar{a}\cdot\bar{i} = x(\bar{i}\cdot\bar{i}) + (\bar{k}\cdot\bar{i})$

$\Rightarrow \bar{a}\cdot\bar{i} = x + 0 = x =$ arbitrary scalar.

28. (2)

Given $\bar{a} = -(2\bar{b} + 4\bar{c})$

Now, $\bar{a}\times\bar{b} = -4(\bar{c}\times\bar{b})$

$= 4(\bar{b}\times\bar{c}) - 2(\bar{c}\times\bar{b}) = 2(\bar{b}\times\bar{c})$

Again $\bar{c}\times\bar{a} = -2(\bar{b}\times\bar{c})$

G.E. = $4(\bar{b}\times\bar{c}) + (\bar{b}\times\bar{c}) + 2(\bar{b}\times\bar{c}) = 7(\bar{b}\times\bar{c})$

29. (4)

$\bar{c} = \bar{a}\times\bar{b} \Rightarrow \bar{c}\perp\bar{a}$ and $\bar{c}\perp\bar{b}$.

Also $\bar{a} = \bar{b}\times\bar{c} \Rightarrow \bar{a}\perp\bar{b}$ and $\bar{a}\perp\bar{c}$

$\Rightarrow$ a, b and c are perpendicular to each other in pairs.

Now $\bar{a}\times\bar{b} - \bar{c} \Rightarrow (\bar{b}\times\bar{c})\times\bar{b} = \bar{c}$ $\quad(\because \bar{a} = \bar{b}\times\bar{c})$

$\Rightarrow (\bar{b}\cdot\bar{b})\bar{c} - (\bar{b}-\bar{c})\bar{b} = \bar{c}$

$\Rightarrow b^2c = c\ (\because \bar{b}\perp\bar{c}$ and $\bar{b}-\bar{c} = 0)$

$\Rightarrow b^2 = 1 \Rightarrow |\bar{b}|^2 = 1$

$\Rightarrow |\bar{b}| = 1 \Rightarrow b = 1(\text{If } |\bar{b}| = \bar{b})$

Also $|\bar{c}| = |\bar{a}\times\bar{b}| = |\bar{a}\cdot 1\cdot\sin(\pi/2)| = |\bar{a}|$

$\Rightarrow \bar{c} = \bar{a}$.

$\therefore$ b = 1 and $\bar{c} = \bar{a}$.

30. (3)

$\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{c} \Rightarrow \bar{a}\cdot(\bar{b}-\bar{c}) = 0$

Or $\bar{b}-\bar{c} = 0$ or $\bar{a}\perp(\bar{b}-\bar{c})$

Again $\bar{a}\times\bar{b} = \bar{a}\times\bar{c} \Rightarrow \bar{a}\times(\bar{b}-\bar{c}) = 0 \Rightarrow 0$

or $\bar{b}-\bar{c} = 0$ or $\bar{a}$ is parallel to $\bar{b}-\bar{c}$

$\Rightarrow$ Since $a \neq 0$ given, we have $\bar{b} = \bar{c}$.

31. (3)

Vector area of the triangle ABC having $A=\bar{a}, B=\bar{b}, C=\bar{c}$ is

$$\frac{1}{2}|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|$$

If p is the length of the ⊥ from A on BC, then the area of the $\Delta ABC=(1/2)BCP$

$$\Rightarrow \frac{1}{2}|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|=\frac{1}{2}|\bar{c}-\bar{b}|P$$

$$\Rightarrow p=|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|\div|\bar{c}-\bar{b}|$$

32. (3)

$$\bar{a}+\bar{b}=3\bar{j}, \bar{b}+\bar{c}=-2\bar{j}+4\bar{j}$$

Now the unit vector ⊥ to $(\bar{a}+\bar{b})$ and $(\bar{b}+\bar{c})$

$$=\frac{(\bar{a}+\bar{b})\times(\bar{b}+\bar{c})}{|(\bar{a}+\bar{b})\times(\bar{b}+\bar{c})|}=\frac{6\bar{k}}{|6\bar{k}|}=\frac{6\bar{k}}{6}=\bar{k}.$$

33. (1)

Given $\bar{a}\times\bar{b}=\bar{c}\times\bar{d}$ …(1)

and $\bar{a}\times\bar{c}=\bar{b}\times\bar{d}$ …(2)

(1) – (2) gives

$$\bar{a}\times(\bar{b}-\bar{c})=(\bar{c}-\bar{b})\times\bar{d}$$

$$\Rightarrow \bar{a}\times(\bar{b}-\bar{c})-(\bar{c}-\bar{b})\times\bar{d}=\bar{0}$$

$$\Rightarrow (\bar{a}-\bar{d})\times(\bar{b}-\bar{c})=\bar{0}$$

$\Rightarrow (\bar{a}-\bar{d})$ is parallel to $\bar{b}-\bar{c}$.

34. (2)

$$\frac{1}{2}|(2\bar{a}+3\bar{b})\times(\bar{b}-\bar{a})|=p|\bar{a}\times\bar{b}|$$

$\Rightarrow \frac{1}{2}|2\bar{a}\times 3\bar{b}+3\bar{a}\times\bar{b}| = p|\bar{a}\times\bar{b}|$

$\Rightarrow \frac{5}{2}|\bar{a}\times\bar{b}| = p|\bar{a}\times\bar{b}|$

$\Rightarrow p = \frac{5}{2}$

35. (1)

Given $\bar{a}\times\bar{b} = 2\bar{a}\times\bar{c}$

$\Rightarrow \bar{a}\times\bar{b} - \bar{a}\times 2\bar{c} = 0$

$\Rightarrow \bar{a}\times(\bar{b}-2\bar{c}) = 0$

Since $\bar{a}\neq 0$, we have $\bar{b}-2\bar{c} = \lambda\bar{a}$

$\Rightarrow (\bar{b}-2\bar{c})^2 = \lambda^2 a^2$

$\Rightarrow |\bar{b}|^2 + 4|\bar{c}|^2 - 4\bar{b}\cdot\bar{c} = \lambda^2|\bar{a}|^2$

where $|\bar{a}| = 1, |\bar{t}| = 1, |\bar{b}| = 4$ (given)

$16 + 4(1) - 4|\bar{b}||\bar{c}|\cos(\bar{b},\bar{c}) = x^2(1)$

$\Rightarrow 20 - 4(4)(1)\cos\left(\cos^{-1}\frac{1}{4}\right) = \lambda^2$

$\Rightarrow \lambda^2 = 20 - 16(1/4) = 16$

$\Rightarrow \lambda = \pm 4$

$\therefore \bar{b} - 2\bar{c} = \pm 4\bar{a}$

36. (2)

Given $\bar{r}\times\bar{a} = \bar{b}\times\bar{a}, \bar{r}\times\bar{b} = \bar{a}\times\bar{b}$

$\Rightarrow \bar{r}\times\bar{a} = -(\bar{r}\times\bar{b}) \Rightarrow (\bar{r}\times\bar{a}) + (\bar{r}\times\bar{b}) = \bar{O}$

$\Rightarrow \bar{r}\times(\bar{a}+\bar{b}) = \bar{O}$

$\Rightarrow \bar{r} = \bar{a}+\bar{b}$

37. (2)

$\bar{a}\cdot\bar{b}=0 \Rightarrow \bar{a}\perp\bar{b}$ and $\bar{a}\cdot\bar{c}=0 \Rightarrow \bar{a}\perp\bar{c}$

$\therefore \bar{a} \parallel \text{to}(\bar{b}\times\bar{c}) \Rightarrow \bar{a}=t(\bar{b}\times\bar{c})$

$\Rightarrow a^2 = t^2(\bar{b}\times\bar{c})^2 = t^2\,|b|^2|c|^2 \sin(\pi/6)$

$\Rightarrow a = t^2(1)(1)(1/4)$

$\Rightarrow 1 = t^2/4 \Rightarrow t = \pm 2$

Hence $\bar{a} = \pm 2(\bar{b}\times\bar{c})$.

38. (3)

Given $\bar{r}\times\bar{b}=\bar{c}\times\bar{b}$

$\Rightarrow (\bar{r}-\bar{c})\times\bar{b}=\bar{O}$

$\Rightarrow (\bar{r}-\bar{c}) \parallel \bar{b} \Rightarrow \bar{r}-\bar{c}=t\bar{b}$ Where t is same scalar

$\bar{r}\cdot\bar{a}=0 \Rightarrow (\bar{c}+t\bar{b})\cdot\bar{a}=0$

$\Rightarrow \bar{c}\cdot\bar{a}+t(\bar{b}\cdot\bar{a})=0 \Rightarrow t=\dfrac{-(\bar{c}\cdot\bar{a})}{\bar{b}\cdot\bar{a}}$

$\therefore \bar{r}=\bar{c}-\left(\dfrac{\bar{c}\cdot\bar{a}}{\bar{b}\cdot\bar{a}}\right)\bar{b}$

39. (2)

We have $\bar{u}\times\bar{v}=|u|\cdot|v|(\sin\theta)n$

Where (u, v) = θ

$u.v=(a-b)(a+b)=a^2-b^2$

$=|a|^2-|b|^2=4-4=0$

$\Rightarrow u\perp v \Rightarrow (u,v)=90°$

$\therefore |u \times v| = |u| \cdot |v| (\sin 90°)$

$(n = u^2 v^2 = (a-b)^2 (a+b)^2$

$= (a^2 + b^2 - 2ab)(a^2 + b^2 + 2a - b)$

$= (8 - 2ab)(8 + 2ab)$

$= 64 - 4(\bar{a} \cdot \bar{b})^2 = 4[16 - (\bar{a} \cdot \bar{b})^2]$

$\Rightarrow |u \times v| = 2\sqrt{16 - (\bar{a} \cdot \bar{b})^2}$

40. (3)

Let $\bar{b} = x\bar{i} + y\bar{j} + z\bar{k}$.

Now $\bar{a} \times \bar{b} = \bar{c}$ and $\bar{a} - \bar{b} = 3$

$\bar{i}(z - y) + \bar{j}(x - z) + \bar{k}(y - x) = \bar{j} - \bar{k}$ and

x + y + z = 3

⇒ z – y = 0, x – z = 1, y – x = –1

and x + y + z = 3

Solving: x = 5/3, y = 2/3, z = 2/3

$\therefore \bar{b} = (5\bar{i} + 2\bar{j} + 2\bar{k})/3$.

41. (1)

Given $\bar{a} \cdot \bar{b} = 0 = \bar{a} - \bar{c}$

$\Rightarrow \bar{a} \perp \bar{b}$ and $\bar{a} \perp \bar{c}$

$\Rightarrow \bar{a}$ is a unit vector perpendicular to both $\bar{b}$ and $\bar{c}$. Also $|\bar{b}| = 1 = |\bar{c}|$

$\therefore \bar{a} = \pm \frac{\bar{b} \times \bar{c}}{|\bar{b} \times \bar{c}|}$

$= \pm \frac{\bar{b} \times \bar{c}}{|\bar{b}||\bar{c}|\sin(\pi/6)} = \pm 2(\bar{b} \times \bar{c})$

42. (3)

$\bar{a}\cdot\bar{b}=\bar{a}\cdot\bar{c}\Rightarrow\bar{a}-(\bar{b}-\bar{c})=\bar{0}$ ...(1)

$\bar{a}\times\bar{b}=\bar{a}\times\bar{c}\Rightarrow\bar{a}\times(\bar{b}-\bar{c})=0$

$(\bar{b}-\bar{c})$ Cannot be both perpendicular and parallel to $\bar{a}\Rightarrow\bar{a}=\bar{0}$ or $\bar{b}-\bar{c}=\bar{0}$

Hence $\bar{a}=\bar{0}$ or $\bar{b}=\bar{c}$.

43. (1)

Let n_1 = a vector normal to the plane of $\bar{i}$ and $\bar{i}+\bar{j}$.

$$\Rightarrow n_1=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 0 & 0\\ 1 & 1 & 0\end{vmatrix}=\bar{k}$$

And let n_2 = a vector $\perp$ to the plane of $\bar{i}-\bar{j}$ and $\bar{j}+\bar{k}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -1 & 0\\ 0 & 1 & 1\end{vmatrix}=-\bar{i}-\bar{j}+\bar{k}$

$a\parallel(n_1\times n_2)$. Now

$$n_1\times n_2=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 0 & 0 & 1\\ -1 & -1 & 1\end{vmatrix}=\bar{i}-\bar{j}\Rightarrow a=\lambda(\bar{i}-\bar{j})\text{ Now,}$$

$$(\bar{a},\bar{i}-2\bar{j}+2\bar{k})=\left[\frac{\lambda(\bar{i}-\bar{j})(\bar{i}-2\bar{j}-2\bar{k})}{\sqrt{\lambda^2+x^2}\sqrt{1+4+4}}\right]$$

$$=\frac{\lambda(1+2)}{\sqrt{2}\lambda(3)}=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^\circ=\frac{\pi}{4}.$$

44.(2)

45.(3)

46.(1)

47.(4)